AP Physics
Chapter 10
Review
1. At $t = 0$, a wheel rotating about a fixed axis at a constant angular acceleration has an angular velocity of $2.0\text{ rad/sec}$. Two seconds later it has turned through $32$ radians. What is the angular acceleration of this wheel?

\[
\text{at } t = 0, \quad \omega_o = 2\text{ rad/sec} \quad \text{at } t = 2\text{ sec}, \quad \Delta \theta = 32\text{ rad}
\]
at \( t = 0, \ \omega_o = 2 \frac{\text{rad}}{\text{sec}} \) at \( t = 2 \text{sec}, \ \Delta \theta = 32 \text{rad} \)

\[ \Delta \theta = \omega_o t + \frac{1}{2} \alpha t^2 \]

\#1

\[ \frac{2(\Delta \theta - \omega_o t)}{t^2} = \alpha \]

\[ \frac{2(32 \text{rad} - (2 \frac{\text{rad}}{\text{sec}})(2 \text{sec}))}{(2 \text{sec})^2} = \alpha \]

\[ 14 \frac{\text{rad}}{\text{sec}^2} = \alpha \]
2. A wheel rotating about a fixed axis has an angular position given by \( \theta = 3 - 2t^3 \), where \( \theta \) is measured in radians and \( t \) in seconds. What is the angular acceleration of the wheel at \( t = 2.0 \) s?

\[
\theta = 3 - 2t^3
\]

\[
\omega = \frac{d\theta}{dt} = -6t^2
\]

\[
\alpha = \frac{d\omega}{dt} = -12t
\]

At \( t = 2 \text{ sec} \), \( \alpha = -24 \text{ rad/sec}^2 \)
3. A wheel rotating about a fixed axis with a constant angular acceleration of $2.0 \, \text{rad/sec}^2$ turns through 16 radians during a 2.0-sec time interval. What is the angular velocity at the end of this time interval?

$$\Delta \theta = \omega_o t + \frac{1}{2} \alpha t^2$$

$$\frac{(\Delta \theta - \frac{1}{2} \alpha t^2)}{t} = \omega_o$$

$$\omega = \omega_o + \alpha t$$

$$\omega = \left(6 \, \text{rad/sec}\right) + \left(2 \, \text{rad/sec}^2\right)(2 \, \text{sec})$$

$$\omega = 10 \, \text{rad/sec}$$

$$\left[16 \, \text{rad} - \frac{1}{2} \left(2 \, \text{rad/sec}^2\right)(2 \, \text{sec})^2\right] = \omega_o$$

$$\frac{6 \, \text{rad}}{2 \, \text{sec}} = \omega_o$$
4. A wheel rotates about a fixed axis with an initial angular velocity of $20 \, \text{rad sec}^{-1}$. During a 5.0 second interval the angular velocity decreases to $10 \, \text{rad sec}^{-1}$. Assume that the angular acceleration is constant during the 5.0 second interval. Through how many radians does the wheel turn during the 5.0 second interval?

\[
\theta = \bar{\omega} t
\]

\[
\theta = \left[ \frac{(20 + 10) \, \text{rad sec}^{-1}}{2} \right] 5 \, \text{sec}
\]

\[
\theta = 75 \text{rad}
\]
5. A thin uniform rod of length $L$ and mass $m$ is pivoted about a horizontal, frictionless pin through one end of the rod. ($I = \frac{1}{3}mL^2$) The rod is released when it makes an angle of $37^\circ$ with the horizontal. What is the angular acceleration of the rod at the instant it is released?

\[
\tau = I\alpha
\]

\[
F_R = I\alpha
\]

\[
mg\cos 37^\circ \frac{L}{2} = \frac{1}{3}mL^2\alpha
\]

\[
g\left(\frac{4}{5}\right)\left(\frac{1}{2}\right) = \frac{1}{3}L\alpha
\]

\[
\frac{2}{5}g = \frac{1}{3}L\alpha
\]

\[
\frac{6g}{5L} = \alpha
\]
6. A wheel rotating about a fixed axis has a constant angular acceleration of \( 4.0 \text{ rad/sec}^2 \). In a 4.0 second interval the wheel turns through an angular displacement of 80 radians. Assuming the wheel started from rest, how long had the wheel been in motion at the start of the 4.0 second interval?

\[
\Delta \theta_{3-2} = \omega_2 t + \frac{1}{2} \alpha t^2
\]

\[
\frac{\Delta \theta_{3-2} - \frac{1}{2} \alpha t^2}{t} = \omega_2
\]

\[
80 \text{ rad} - \frac{1}{2} \left( 4 \text{ rad/sec}^2 \right) (4 \text{ sec})^2
\]

\[
= \omega_2
\]

\[
\frac{80 \text{ rad} - 12 \text{ rad/sec}}{4 \text{ sec}} = \omega_2
\]

\[
\omega_2 = \omega_o + \alpha t
\]

\[
\omega_2 = t
\]

\[
\alpha = \frac{t}{4 \text{ rad/sec}^2} = 12 \text{ rad/sec}
\]

\[
12 \text{ rad/sec} = \omega_2
\]

8 sec
7. A mass \((m_1 = 5.0 \text{ kg})\) is connected by a light cord to a mass \((m_2 = 4.0 \text{ kg})\) which slides on a smooth surface, as shown in the figure. The pulley (radius = 0.20\(m\)) rotates about a frictionless axle. The acceleration of \(m_2\) is \(3.5 \frac{m}{\text{sec}^2}\). What is the moment of inertia of the pulley?
\[ F_{\text{Net}} = ma \]
\[ m_1g - T_1 + T_2 = (m_1 + m_2)a \]
\[ m_1g - (T_1 - T_2) = (m_1 + m_2)a \]
\[ m_1g - \left( \frac{Ia}{R^2} \right) = (m_1 + m_2)a \]
\[ m_1g - (m_1 + m_2)a = \frac{Ia}{R^2} \]
\[ R^2 \left( m_1g - (m_1 + m_2)a \right) \]
\[ a \]
\[ 0.04m^2 \left( 49N - 31.5N \right) \]
\[ \frac{3.5 \frac{m}{\text{sec}^2}}{I} \]
\[ I = 0.20 Kg \cdot m^2 \]
8. A wheel (radius = 0.25 m) is mounted on a frictionless, horizontal axis. The moment of inertia of the wheel about the axis is $0.040\,Kg \cdot m^2$. A light cord wrapped around the wheel supports a 0.50$Kg$ object as shown in the figure. The object is released from rest. What is the magnitude of the acceleration of the 0.50$Kg$ object?
\[ F_{Net} = ma \]
\[ mg - T = ma \]
\[ mg - \frac{Ia}{R^2} = ma \]
\[ mg = \left( m + \frac{I}{R^2} \right)a \]

\[
\frac{mg}{\left( m + \frac{I}{R^2} \right)} = a
\]

\[
\frac{0.5 \text{Kg} \left( 9.8 \frac{m}{\text{sec}^2} \right)}{0.5 \text{Kg} + \frac{0.04 \text{Kg} \cdot m^2}{0.0625 m^2}} = a
\]

\[ 4.3 \frac{m}{\text{sec}^2} = a \]
9. A wheel rotating about a fixed axis with a constant angular acceleration of $2 \, \text{rad/sec}^2$ starts from rest at $t = 0$. The wheel has a diameter of 20 cm. What is the magnitude of the total linear acceleration of a point on the outer edge of the wheel at $t = 0.60$ s?
\[ a_t = R\alpha = (0.1\text{ m})\left(2 \text{ \(\frac{\text{rad}}{\text{sec}^2}\)}\right) = 0.2 \text{ \(\frac{m}{\text{sec}^2}\)} \]

\[ \omega = \alpha \ t = \left(2 \text{ \(\frac{\text{rad}}{\text{sec}^2}\)}\right)(0.6 \text{ sec}) = 1.2 \text{ \(\frac{\text{rad}}{\text{sec}}\)} \]

\[ a_R = R\omega^2 = (0.1\text{ m})\left(1.2 \text{ \(\frac{\text{rad}}{\text{sec}}\)}\right)^2 = 0.144 \text{ \(\frac{m}{\text{sec}^2}\)} \]

\[ a = \sqrt{a_t^2 + a_R^2} = \sqrt{\left(0.2 \text{ \(\frac{m}{\text{sec}^2}\)}\right)^2 + \left(0.144 \text{ \(\frac{m}{\text{sec}^2}\)}\right)^2} \]

\[ a = 0.25 \text{ \(\frac{m}{\text{sec}^2}\)} \]
10. A cylinder rotating about its axis with a constant angular acceleration of $1.6 \ \frac{\text{rad}}{\text{sec}^2}$ starts from rest at $t = 0$. At the instant when it has turned through $0.40$ radian, what is the magnitude of the total linear acceleration of a point on the rim (radius = 13 cm)?

\[
\begin{align*}
\theta &= \frac{1}{2} \alpha t^2 \\
\sqrt{\frac{2\theta}{\alpha}} &= t = \sqrt{\frac{2(0.4 \text{ rad})}{1.6 \ \frac{\text{rad}}{\text{sec}^2}}} \\
t &= 0.707 \ \text{sec}
\end{align*}
\]

\[
\omega &= \alpha t \\
\omega &= (1.6 \ \frac{\text{rad}}{\text{sec}^2})(0.707 \ \text{sec}) \\
\omega &= 1.13 \ \frac{\text{rad}}{\text{sec}}
\]
10 What is the magnitude of the total linear acceleration of a point on the rim?

\[ a_t = R\alpha = (0.13\text{m})(1.6\text{ rad}) = 0.208\text{ m/sec}^2 \]

\[ a_R = R\omega^2 = (0.13\text{m})(1.13\text{ rad/sec})^2 = 0.166\text{ m/sec}^2 \]

\[ a = \sqrt{a_t^2 + a_R^2} = \sqrt{(0.208\text{ m/sec}^2)^2 + (0.166\text{ m/sec}^2)^2} \]

\[ a = 0.27\text{ m/sec}^2 \]
11. A horizontal disk with a radius of 10 cm rotates about a vertical axis through its center. The disk starts from rest at \( t = 0 \) and has a constant angular acceleration of \( 2.1 \frac{\text{rad}}{\text{sec}^2} \). At what value of \( t \) will the radial and tangential components of the linear acceleration of a point on the rim of the disk be equal in magnitude?

\[
\begin{align*}
  a_R &= a_t \\
  R\omega^2 &= R\alpha \\
  \alpha^2 t^2 &= \alpha \\
  \alpha t^2 &= 1 \\
  t &= \sqrt{\frac{1}{\alpha}} = \sqrt{\frac{1}{2.1 \frac{\text{rad}}{\text{sec}^2}}} \\
  t &= 0.69 \text{ sec}
\end{align*}
\]
12. A mass $m = 4.0 \text{ kg}$ is connected, as shown, by a light cord to a mass $M = 6.0 \text{ kg}$, which slides on a smooth horizontal surface. The pulley rotates about a frictionless axle and has a radius $R = 0.12 \text{ m}$ and a moment of inertia $I = 0.090 \text{ Kg} \cdot \text{m}^2$. The cord does not slip on the pulley. What is the magnitude of the acceleration of $m$?
\[ F_{Net} = ma \]

\[ mg - \frac{Ia}{R^2} = (m + M)a \]

\[ mg = \left( m + M + \frac{I}{R^2} \right)a \]

\[ 12 \left( \frac{m}{m + M + \frac{I}{R^2}} \right) g = a \]

\[ \frac{4Kg}{\left(4Kg + 6Kg + \frac{0.09Kg \cdot m^2}{(0.12m)^2} \right)} \cdot 9.8 \frac{m}{\text{sec}^2} = a \]

\[ 2.41 \frac{m}{\text{sec}^2} = a \]
13. Two particles \((m_1 = 0.20 \text{ kg}, \ m_2 = 0.30 \text{ kg})\) are positioned at the ends of a 2.0-m long rod of negligible mass. What is the moment of inertia of this rigid body about an axis perpendicular to the rod and through the center of mass?

\[
l = 2m
\]
\[ x_{\text{CofM}} = \frac{\sum mx}{\sum m} = \frac{(0.2\text{Kg})(0) + (0.3\text{Kg})(2m)}{0.5\text{Kg}} \]

\[ x_{\text{CofM}} = 1.2m \text{ from the 0.2Kg mass} \]

\[ x_{\text{CofM}} = 0.8m \text{ from the 0.3Kg mass} \]

\[ I = \sum mR^2 = 0.2\text{Kg}(1.2m)^2 + 0.3\text{Kg}(0.8m)^2 \]

\[ I = (0.288 + 0.192)\text{Kg} \cdot m^2 \]

\[ I = 0.48\text{Kg} \cdot m^2 \]
14. Four identical particles (mass of each = 0.5 kg) are placed at the vertices of a rectangle (2.0 m by 3.0 m) and held in those positions by four light rods which form the sides of the rectangle. What is the moment of inertia of this rigid body about an axis that passes through the center of mass of the body and is perpendicular to the page?

\[
I = \sum mR^2 = 4(0.5Kg)\left(\sqrt{3.25m}\right)^2 = 6.5Kg \cdot m^2
\]
15. The rigid object shown is rotated about an axis perpendicular to the paper and through point $P$. The total kinetic energy of the object as it rotates is equal to $K$. If mass $= M$ and length $= L$, what is the angular velocity of the object? Neglect the mass of the connecting rods and treat the masses as particles.

\[ K = \frac{1}{2} I \omega^2 \]

\[ \frac{2K}{I} = \omega^2 \]

\[ \frac{2K}{2(ML^2) + 2(2M)\left(\frac{L}{2}\right)^2} = \omega^2 \]

\[ \frac{2K}{3ML^2} = \omega^2 \]

\[ \frac{1}{L} \sqrt{\frac{2K}{3M}} = \omega \]
\[ x_{CofM} = \frac{\sum mx}{\sum m} = \frac{ML + 3M(2L)}{5M} = \frac{7}{5}L \text{ from the left edge.} \]

\[ I = \sum mR^2 = M\left(\frac{7}{5}L\right)^2 + M\left(\frac{2}{5}L\right)^2 + 3M\left(\frac{3}{5}L\right)^2 = \frac{16}{5}ML^2 \]

16. If the mass of each connecting rod shown above is negligible, what is the moment of inertia about an axis perpendicular to the paper through the center of mass?
17. A uniform rod of mass $M$ and length $L$, lying on a frictionless horizontal plane, is free to pivot about a vertical axis through one end, as shown. The moment of inertia of the rod about this axis is given by $\frac{1}{3}ML^2$. If a force $F$ is applied at some angle $\theta$, and acts as shown, what is the resulting angular acceleration about the pivot point?

\[ \sum \tau = I\alpha \]

\[ \sum F \perp R = I\alpha \]
\[ F \sin \theta L = \frac{1}{3} M L^2 \alpha \]
\[ F \sin \theta = \frac{1}{3} M L \alpha \]
\[ 3 \ F \sin \theta = M L \alpha \]
\[ \frac{3F \sin \theta}{ML} = \alpha \]
A uniform rod of mass $m$ and length $L$ is free to rotate about a frictionless pivot at one end. The rod is released from rest in the horizontal position. What is the angular acceleration of the rod at the instant it is $60^\circ$ below the horizontal?
19. A uniform meter stick is pivoted to rotate about a horizontal axis through the 25-cm mark on the stick. The stick is released from rest in a horizontal position. The moment of inertia of a uniform rod about an axis perpendicular to the rod and through the end of the rod is given by \( \frac{1}{3} ML^2 \). Determine the magnitude of the initial angular acceleration of the stick.
\[ I = \sum mR^2 = \frac{1}{3} \left( \frac{1}{4} M \right) \left( \frac{L}{4} \right)^2 + \frac{1}{3} \left( \frac{3}{4} M \right) \left( \frac{3L}{4} \right)^2 \]

\[ I = \frac{1}{12} M \left( \frac{L^2}{16} \right) + \frac{1}{4} M \left( \frac{9L^2}{16} \right) = \frac{1}{12} M \left( \frac{L^2}{16} \right) + \frac{27}{12} M \left( \frac{L^2}{16} \right) \]

\[ I = \frac{28}{12} M \left( \frac{L^2}{16} \right) = \frac{7}{48} ML^2 \]

\[ I = \int_{-\frac{L}{4}}^{\frac{3L}{4}} \frac{M}{L} x^2 \, dx = \frac{M}{L} \left[ \frac{1}{3} x^3 \right]_{-\frac{L}{4}}^{\frac{3L}{4}} = \frac{M}{L} \left[ \frac{27L^3}{3 \cdot 64} + \frac{L^3}{3 \cdot 64} \right] \]

\[ I = \frac{28}{192} ML^2 = \frac{7}{48} ML^2 \]
\[ \tau = I\alpha \]
\[ F_{\perp}R = I\alpha \]
\[ Mg\frac{L}{4} = \frac{7}{48}ML^2\alpha \]
\[ g = \frac{7}{12}L\alpha \]
\[ \frac{12g}{7L} = \alpha \]

19.3
20. A uniform rod of length \( L \) and mass \( M \) is pivoted about a horizontal frictionless pin through one end. The rod is released from rest at an angle of 30° below the horizontal. What is the angular speed of the rod when it passes through the vertical position?

\[
U_{Top} = K_{Bottom}
\]

\[
mg \frac{L}{4} = \frac{1}{2} I \omega^2
\]

\[
mg \frac{L}{4} = \frac{1}{2} \left( \frac{1}{3} mL^2 \right) \omega^2
\]

\[
g \frac{1}{2} = \frac{1}{3} L \omega^2
\]

\[
\sqrt{\frac{3g}{2L}} = \omega
\]
21. The figure below shows a graph of angular velocity as a function of time for a car driving around a circular track. Through how many radians does the car travel in the first 10 minutes?

\[ \left[ (20 + 40 + 10) \frac{rad \cdot \text{min}}{\text{sec}} \right] \left( \frac{60 \text{ sec}}{\text{min}} \right) = 4200 \text{rad} \]
22. What is the total Kinetic Energy of a 10 Kg solid sphere whose radius 25 cm and is rolling across a horizontal floor at

\[ K_{Net} = K + K_\theta \]

\[ K_{Net} = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 \]

\[ K_{Net} = \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{2}{5}mR^2\right)\frac{v^2}{R^2} \]

\[ K_{Net} = \frac{7}{10}mv^2 = \frac{7}{10}(10\text{Kg})(2\text{m/sec})^2 \]

\[ K_{Net} = 28J \]
23. A 2 Kg, hollow sphere is rolling on a horizontal surface with no slippage. What is the ratio of the sphere’s rotational Kinetic energy to its translational kinetic energy?

\[
\frac{1}{2} I \omega^2 = \frac{2}{3} mR^2 \frac{v^2}{R^2} = \frac{2}{3}
\]

\[
\frac{1}{2} mv^2 = \frac{2}{3} mv^2 = \frac{2}{3}
\]
24. The graphs below show angular velocity as a function of time. In which one is the magnitude of the angular acceleration constantly decreasing?

A) ![Graph A]

B) ![Graph B]

C) ![Graph C]

D) ![Graph D]

E) ![Graph E]
25. The figure below shows a graph of angular velocity versus time for a woman bicycling around a circular track. What is her angular displacement (in rad) in the first 8 minutes?

\[
\Delta \theta = \left( 4\pi \text{ rad/min} \right) \left( 8 \text{ min} \right) \quad \frac{1}{2} \\
\Delta \theta = 16\pi \text{ rad}
\]
26. What is the Rotational Kinetic Energy of a solid sphere of mass 5 \( kg \) and radius 0.5 m if it is rotating about its diameter at a constant rate of 6 \( \frac{rad}{sec} \)?

\[
K_\theta = \frac{1}{2} I \omega^2
\]

\[
K_\theta = \frac{1}{2} \left(\frac{2}{5} mR^2\right) \omega^2
\]

\[
K_\theta = \frac{1}{5} (5Kg)(0.5m)^2 \left(6 \frac{rad}{sec}\right)^2
\]

\[
K_\theta = 9 J
\]
27. Two Forces of magnitude 50 N, as shown in the figure below, act on a cylinder of radius 4 m and mass 6.25 kg. The cylinder, which is initially at rest, sits on a frictionless surface. After 1 second, the velocity and angular velocity of the cylinder in $\text{m/sec}$ and $\text{rad/sec}$ are respectively:

\[ \tau = I \alpha \]

\[ FR = I \alpha \]

\[ \frac{FR}{\frac{1}{2} mR^2} = \alpha \]

\[ \frac{2F}{mR} = \alpha \]

\[ \frac{2(50N)}{6.25Kg(4m)} = \alpha \]

\[ 4 \ \text{rad/sec}^2 = \alpha \]

\[ \omega = \alpha t \]

\[ \omega = 4 \ \text{rad/sec}^2 (1 \text{sec}) \]

\[ \omega = 4 \ \text{rad/sec} \]
28. Two blocks, \( m_1 = 1 \text{ kg} \) and \( m_2 = 2 \text{ kg} \), are connected by a light string as shown in the figure. If the radius of the pulley is 1 m and its moment of inertia is \( 5 \text{ Kg} \cdot \text{m}^2 \), what fraction of \( g \) is the acceleration of the system?
\[ F_{\text{Net}} = ma \]

\[ m_2 g - T_2 + T_1 - m_1 g = (m_1 + m_2)a \]

\[ (m_2 - m_1)g - \frac{Ia}{R^2} = (m_1 + m_2)a \]

\[ (m_2 - m_1)g = \left( m_1 + m_2 + \frac{I}{R^2} \right)a \]

\[ \frac{(m_2 - m_1)g}{m_1 + m_2 + \frac{I}{R^2}} = a \]

\[ \frac{1}{8} g = a \]

\[ I = 5 \text{Kg} \cdot m^2 \]

\[ m_1 = 1 \text{ kg} \]

\[ m_2 = 2 \text{ kg} \]
29. The figure below shows a graph of angular velocity versus time for a man bicycling around a circular track. What is his average angular acceleration in the first 10 minutes?

\[
\alpha = \text{slope} = \frac{(-4\pi - 4\pi) \text{ rad}}{\text{sec}} = \frac{-8\pi}{600} \text{ rad} \cdot \frac{10 \text{ min}}{60 \text{ sec}} = -\frac{\pi}{75} \text{ rad/sec}^2
\]
A mass $m$ is moving with a speed $v_o$ as it moves left to right across the page as shown below. At some time $t$, the mass explodes into two pieces with a piece $\frac{1}{5}m$ moving to the left with a velocity $-2v_o$.

30. What is the velocity of the second piece?

\[
mv_o = -\frac{2}{5}mv_o + \frac{7}{5}mv_o
\]

Substituting $p = \frac{7}{5}mv_o$ and dividing by $m$, we get:

\[
v' = \frac{7}{4}v_o
\]

31. What is the velocity of the center of mass after the explosion?

\[
v_{CofM} = \frac{\sum mv}{\sum m} = \frac{-\frac{2}{5}mv_o + \frac{7}{5}mv_o}{m} = v_o
\]
32. The Net Torque on the following system is most nearly.

\[ \tau_{Net} = \sum F \times R = 20N(0.5m) - (7N + 5N)(1.5m) \]

\[ \tau_{Net} = -8Nm \]
33. A tiered-solid disk of mass $M$ is free to rotate on a frictionless axis. Two masses are hung over the disk so that it makes an Atwood's machine. Someone tells you the inner radius is half as large as the outer radius, and that the moment of inertia of the disk is $35Kg \cdot m^2$. If the left hand mass is 4 Kg, what value of mass $m$ would keep the system in static equilibrium?

$$\tau_{CCW} = \tau_{CW}$$

$$m_L g (2R) = m_R g R$$

$$2(4 Kg) = m_R$$

$$8 Kg = m_R$$